A day of control on PDEs

Thursday October 27 2022

Université Polytechnique Hauts-de-France (Valenciennes)

Laboratoire de Matériaux Céramiques et de Mathématiques - CERAMATHS

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Program and Talks

- **8h30-8h45** : Registration
- **8h45-9h00** : Opening Talks
- **9h00-9h45** :

Christophe Prieur

Université Grenoble Alpes, CNRS

Stabilization of Nonlinear PDE by Means of Nonlinear Boundary Controls

In this presentation, the focus will be done on the design of boundary controls for distributed parameter systems as those described by linear and nonlinear partial differential equations. Saturated controllers will be discussed in this talk as those modeling feedback laws in presence of amplitude constraints. We will review some techniques for the stability analysis and the derivations of design conditions for various PDEs as parabolic and hyperbolic ones. An application in nuclear fusion will conclude this lecture.

- **10h00-10h45** :

Virginie Régnier

Université Polytechnique Hauts-de-France, CERAMATHS, Valenciennes-France

Do Shape Memory Alloy cables restrain the vibrations of girder bridges ? - a mathematical point of view

The aim is to study the energy decay of a damped Euler-Bernoulli beam which is subject to a pointwise feedback force representing a Shape Memory Alloy (SMA) cable. The considered problem is that of the paper by A-R. Liu, C-H. Liu, J-Y. Fu, Y-L. Pi, Y-H. Huang and J-P. Zhang [1] but, for simplicity, the modelization does not take into account the additional stiffness term they considered. An explicit expression is given for the resolvent of the underlying operator as well as its eigenvalues and eigenfunctions. The exponential decay of the energy is established. The fastest decay rate is given by the supremum of the real part of the spectrum of the infinitesimal generator of the underlying semigroup since the existence of a Riesz basis is proved. To the question "Do Shape Memory Alloy cables restrain the vibrations of girder bridges ?", the experiments in [1] answer positively. This mathematical study does not allow to give a definite answer yet. The only presence of these cables may not be enough. Some physical parameters have to be chosen carefully.

References

[1] A-R. Liu, C-H. Liu, J-Y. Fu, Y-L. Pi, Y-H. Huang, J-P. Zhang, A Method of Reinforcement and Vibration Reduction of Girder Bridges Using Shape Memory Alloy Cables, Int. J. Struct. Stab. Dyn. **17** (No. 7) (2017) 1750076.

- **11h00-11h30** : Coffee Break.

- **11h30-12h15** :

Genni Fragnelli

University of Tuscia-Italy

Degenerate wave equations with a drift term : new results on controllability and stabilization

We consider the problem

$$\begin{cases} u_{tt} - a(x)u_{xx} - b(x)u_x = 0, & (t,x) \in Q, \\ u(t,0) = 0, & t \in [0,+\infty), \\ u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x), & x \in (0,1), \end{cases} \tag{1}$$

where $Q = (0,+\infty) \times (0,1)$, $f \in L^2_{\text{loc}}[0,+\infty)$, $a, b \in C^0([0,1])$, $a > 0$ on $(0,1]$ and $a(0) = 0$. At $x = 1$ we consider different boundary conditions according to the considered problem. If we are interested in a controllability problem (see [2]) we assume

$$u(t,1) = f(t), \quad t \in [0,+\infty);$$

thus the function f acts as a boundary control and it is used to drive the solution to 0 at a given time T.

Otherwise, if we are interested in the stabilization problem (see [3]) we consider as a boundary condition the following damping one

$$u_t(t,1) + \eta u_x(t,1) + \beta u(t,1) = 0, \quad t \in [0,+\infty),$$

where η is a given function and β is a nonnegative constant. Clearly the presence of the drift term leads us to use different spaces with respect to the ones in [1] and it gives rise to some new difficulties. However, thanks to some suitable assumptions on the drift term, one can prove some estimates on the associated energy that are crucial to drive the solution to 0 at time T or to obtain a uniform exponential decay.

References

[1] F. Alabau-Boussouira, P. Cannarsa and G. Leugering, Control and stabilization of degenerate wave equations, SIAM J. Control Optim., **55** (2017), pp. 2052-2087.

[2] I. Boutaayamou, G. Fragnelli and D. Mugnai, *Boundary controllability for a degenerate wave equation in non divergence form with drift*, submitted.

[3] G. Fragnelli and D. Mugnai, *Linear stabilization for a degenerate wave equation in non divergence form with drift*, preprint.

- **12h30-14h00** : Lunch Break.

• **14h00-14h45** :

Grégoire Allaire

Centre de Mathématiques Appliquées, École Polytechnique, Palaiseau- France

Concurrent optimization of shape and scanning path in additive manufacturing

We study the concurrent path planning optimization and shape optimization for a mechanical structure built by a powder bed fusion additive manufacturing process. Structures are often optimized for their mechanical performance alone but rarely in a combined way with their path planning building process. In this work, a two dimensional model (in the layer plane) of the process is proposed under a steady state assumption. Then a systematic path optimization approach, free from a priori restrictions, is coupled to a structural optimization tool, both of them based on shape optimization theory. The typical optimization problem is twofold. First, the desired structure has to be melt, without over-heating (to avoid thermally induced residual stresses) and possibly with a minimal path length. The first state equation is the heat equation with a source term depending on the scanning path. Second, the structure should be of minimal compliance for maximal rigidity. The second state equation is the linearized elasticity system. Both state equations are numerically solved by finite elements. Shape derivatives are computed by the Hadamard method for both optimization variables, the path and the structural shape. Gradient type algorithms are deduced (in a level set framework) and tested on 2-d examples. This multiphysics optimization leads to innovative and promising results. First, they confirm that it is essential to take into account the part shape in the scanning path optimization. Second, they also give hints to some design recipes : for given material and heat source parameters, the thickness of a bar is a key ingredient which determines the type of path pattern to scan it : straight line, Omega-pattern or Wave-pattern. Some extensions to the unsteady case are discussed : the manufacturing time is minimized under two constraints : melting the required structure and avoiding any over-heating causing thermally induced residual stresses.

• **15h00-15h45** :

Kaïs Ammari

Faculty of Sciences of Monastir, Lab Analysis and Control of PDEs-Tunisia

Dispersion on regular trees

In this talk, we prove dispersive estimates for two models : the adjacency matrix on a discrete regular tree, and the Schrödinger equation on a metric regular tree with the same potential on each edge/vertex. The latter model can be thought of as an extension of the case of periodic Schrödinger operators on the real line. We establish a $t^{-3/2}$-decay for both models which is sharp, as we give the first-order asymptotics. This is a joint work with Mostafa Sabri.

• **16h00-16h30** : Coffee Break.

• **16h30-17h15** :

Ali Wehbe

Lebanese University, KALMA Laboratory-Lebanon

Stability for an interface transmission problem of wave-plate equations with dynamical boundary controls

We investigate a two-dimensional transmission model consisting of a wave equation and a Kirchoff plate equation with dynamical boundary controls under geometric conditions. The two equations are coupled through transmission conditions along a steady interface between the domains in which the wave and the plate equations evolve respectively. Our primary concern is the stability analysis of the system which has not appeared in the literature. For this aim, using a unique continuation theorem, the strong stability of the system is proved without any geometric condition and in the absence of compactness of the resolvent. However, using frequency domain approach from the semigroup theory which combines a contradiction argument with multiplier technique, we establish a polynomial energy decay estimate of type t^{-1} for smooth initial data. This method leads to certain geometrical conditions concerning the wave's and the plate's domains.