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Seminar presentation abstract

Introduction to Isogeometric Analysis and its efficiency for solving Laplace eigenvalue problems

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Isogeometric Analysis (IGA) is a technique for the discretization of Partial Differential Equations (PDEs) which uses the same class of functions most commonly used in Computer Aided Design, namely Non-Uniform Rational B-Splines (NURBS), for the exact description of the computational domain geometry and for the PDE solution space [1]. As NURBS spaces include as a special case the piece-wise polynomial spaces usually used in the Finite Element Method (FEM), IGA can be understood as a generalization of standard FEMs where more regular functions can be employed. This higher regularity has been shown to lead to some advantages of IGA over FEM, e.g., a better approximation of the eigenvalues of the Laplacian on rectangular domains [2, 4]. On the one hand, this is due to the fact that finite elements produce so-called optical branches, meaning that the upper part of the spectrum is approximated inaccurately, a phenomenon which does not appear when using isogeometric elements of maximal smoothness. On the other hand, the overall accuracy is improved as the approximation constant of maximally smooth spline spaces is superior to the one obtained with classical finite element spaces [5].

We examine how the improved eigenvalue approximation can be extended to certain domains with singularities. Therefore, we consider the Laplace eigenvalue problem on a circular sector with a conical point in the center as our model problem. Some of the resulting eigenfunctions are not smooth but have a singularity of type r^ν, thus the corresponding eigenvalues can not be approximated well with uniform refinement procedures. Mesh grading techniques help to recover the optimal convergence order and have been proven effective for multipatch discontinuous Galerkin IGA schemes [3]. We introduce a single patch continuous Galerkin mesh grading approach for our model domain and prove optimal convergence order for the eigenfunctions and eigenvalues without using discontinuous Galerkin. Finally, we use maximally smooth NURBS of higher degrees in combination with our approach and compare the spectral approximation accuracy to classical higher order finite element graded mesh schemes. To conclude the presentation, we show how the powerful spectral approximation properties of IGA can be used for an application to shape identifcation.

Key Words: Isogeometric Analysis, Eigenvalue approximation, Mesh grading, Singularities.

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