ON THE COMPACTNESS OF THE DE RHAM DIAGRAM: CONTINUOUS AND DISCRETE

SIMON LEMAIRE, INRIA LILLE

The L^2 de Rham diagram (here below in 3D)

$$\{0\} \xrightarrow{0} H^1 \xrightarrow{\textbf{grad}} \boldsymbol{H}(\textbf{curl}) \xrightarrow{\textbf{curl}} \boldsymbol{H}(\text{div}) \xrightarrow{\text{div}} L^2 \xrightarrow{0} \{0\}$$

is a prototypical example of a Hilbert complex, that is, a sequence of Hilbert spaces connected through differential operators which satisfies the complex property (the image of a given operator in the sequence is contained in the kernel of the next). Discretizations of the de Rham diagram that preserve the complex property are referred to as *compatible*, and are desirable in practice for a number of reasons. In the finite element (FE) setting on standard meshes, the Lagrange - Nédélec - Raviart–Thomas sequence is a well-known example of such a compatible discretization. In this talk, we are more particularly interested in the compactness properties of the de Rham diagram. At the continuous level, these compactness results are due to Rellich (for H^1), and to Weck/Weber (for $\boldsymbol{H}(\textbf{curl})$ and $\boldsymbol{H}(\text{div})$). We investigate here the validity of such results at the discrete level, starting from the FE setting, then covering the cases of two different polytopal methods (that is, numerical methods capable of accommodating general polytopal meshes), one being compatible and the other not. The material of this talk is based on joint works (i) with T. Chaumont-Frelet (INRIA Lille) and J. Droniou (CNRS, Univ. Montpellier) on the one hand, and (ii) with S. Pitassi (Univ. Montpellier) on the other hand.